7N-05-CR

30133

JWOPT — An Optimization and Analysis Program for Joined Wing and Conventional Aircraft Structures

John W. Gallman
Prof. Ilan Kroo
January 1988

Department of Aeronautics and Astronautics
Stanford University
Stanford CA 94305

Abstract

Interest in the application of the joined wing aircraft configuration[1] has created a need for design and analysis tools that enable quick, yet accurate, assessment of the configuration's advantages. The program JWOPT was written to study the effects of geometric parameters on the drag and structural weight of joined wing aircraft. JWOPT is an aerodynamic and structural optimization program which is capable of analyzing two-surface aircraft configurations, including joined wings. Extended lifting line theory and inextensible beam analyses are used to provide comparisons of weight and drag for stable, trimmed, joined wing and conventional designs. This paper is intended as a user's manual for the program JWOPT. It includes an explanation of the overall optimization procedure, a description of each major computational section, and an operating manual with sample input and output files.

(NASA-CR-197018) JWOPT: AN OPTIMIZATION AND ANALYSIS PROGRAM FOR JOINED WING AND CONVENTIONAL AIRCRAFT STRUCTURES (Stanford Univ.) 37 p

N95-70298

Unclas

Z9/05 0030133

Nomenclature

A	Cross-sectional area of box beam.
C_L	Aircraft total lift coefficient.
C_{Lref}	Reference lift coefficient.
C_M	Aircraft pitching moment coefficient.
Cref	Reference wing chord (Cref = Sref/bref).
Cx	One half the beam width in inches.
Cz	One half the beam hight in inches.
bref	Reference wing span.
Ix	Beam moment of inertia (in.^{4}) about local x-axis.
Iz	Beam moment of inertia (in.^{4}) about local z-axis.
J	Torsional moment of inertia (in.^{4}) about local y-axis.
Mx	Moment (lb.-in.) about local x-axis.
Mz	Moment (lb.-in.) about local z-axis.
M_∞	Free steam Mach Number.
sm	Aircraft static margin.
Sref	Reference wing area. One half the wetted area of the wing and tail, where the wing and tail are assumed to be flat plates.
tx	Beam web thickness in inches.
tz	Beam cap thickness in inches.
U_∞	Freestream velocity
Xdel	Initial increment in Xtotal during wing positioning.
Xtotal	Distance between the wing and tail root quarter chords in the global x-direction.
ΔR	Joint deflections and rotations in global coordinates due to reaction forces only.
ΔT	Deflections and rotations of the tail at the tip (joint location).
ΔW	Deflections and rotations of the wing at the joint
σ	Normal stress at a given spanwise station.

<u>Subscripts</u>

a	Applied loads
r	Reaction forces
req	Required
max	Maximum

Introduction

It has been suggested[1-4] that the joined wing configuration, (see Fig. 1),proposed by J. Wolkovitch[1], offers several advantages when compared with conventional designs. Possible increases in maximum lift capability, reductions in induced drag, and savings in structural weight have been predicted for several different joined wing geometries. The purpose of this program is to provide a means for rapidly assessing potential reductions in cruise drag and structural weight.

A computer program called JWOPT was written to enable the aerodynamic and structural optimization of a wide range of joined wing configurations. The aerodynamic loads used for structural design within the program are calculated for each case using extended lifting line theory. These aerodynamic loads are then used to optimize a box beam structure based on a maximum normal stress constraint.

This paper describes the overall program organization, each of the major computational subroutines, and the input and output formats. The subroutines include (1) LASUB which use extended lifting line theory[7] to calculate the aerodynamic loads, (2) PLANF which uses simple input data such as wing sweep, taper ratio and span to determine the wing position which provides the required static margin, (3) TLOAD which uses the loads calculated by LASUB to determine the aerodynamic loading for minimum trimmed induced drag, (4) BEAM which calculates the distributed structural loads by integrating the beam deflection equations, and (5) subroutines which calculate the beam moments of inertia and optimizes its dimensions for minimum weight.

Computational Procedures

The distance between the vertical fin, which serves as the attachment point for the tail, and the center of gravity positions are fixed when considering joined wing variations. Since the wings are joined and the root of the tail is fixed to the vertical fin, the sweep and dihedral of the tail are determined by the horizontal wing position which gives the desired static margin. Therefore wing sweep, dihedral, the ratio of tail and wing aspect ratios, and joint location are the design variables used to determine the configuration with the least drag and structural weight.

Program Structure

The main routine, JWOPT, organizes the geometric, aerodynamic, and structural subprograms and contains the optimization algorithm for structural weight. A flow chart which displays the sequence of calculations and the overall structure of JWOPT is shown in Fig. 2. A more detailed flow-chart which includes all of the program subroutines is shown in Fig. 3. The structural optimization loop is highlighted in Fig. 3 with bold lines. As shown in Fig. 2 the program begins by reading the input data file. The subroutine PLANF is then called to determine the wing position which provides the required static margin and calculates other geometric parameters. TLOAD is then used to calculate the aerodynamic load distributions which trim the aircraft at $C_L = C_{Lref}$ and produce minimum induced drag. The subroutine LASUB is used by TLOAD to calculate aerodynamic loadings using extended lifting line theory. The first step in the structural optimization loop is to compute the cross-sectional area and moment of inertia distributions based on input data or the most recent beam dimensions provided by the structural optimizer (see Fig. 2). The structural weight is now calculated by integrating the cross-sectional area distribution along the span to get structural volume and multiplying by the material density. A test is now made to see if the weight has converged to a minimum value. If the two most recent iterations produce wing and tail weights which have changed less than a specified weight tolerance, then JWOPT exits the optimization loop and prints the desired results. If the weight has not converged to within the desired tolerance, subroutine BEAM is called to calculate the shear and moment distributions. Given the shear and moment distributions the structural optimizer determines the beam cap and web thicknesses which satisfy the maximum allowable stress constraint and produce a minimum weight structure. The optimization loop now begins again by recalculating cross-sectional areas and moments of inertia, based on these new dimensions. Detailed descriptions will now be given for each of the primary analysis sections.

Subroutine LASUB

same address as Ref. 9

LASUB is a subroutine version of the program LinAir published by Desktop Aeronautics (Ref. 10). This subroutine calculates the forces, moments, and lift distributions on multi-element, nonplanar lifting surfaces using a discrete vortex Weissenger method. Forces and moments are computed using Trefftz plane induced velocities.

LinAir solves the Prandtl-Glauert equation, the linear partial differential equation describing inviscid, irrotational, subsonic flow:

$$(1 - M_\infty^2)\, U_x + V_y + W_z = 0$$

with U, V, and W the three components of the flow velocity in the x, y, and z directions respectively and M_∞, the freestream Mach number.

This linear equation is solved by superposition of known solutions, namely discrete line vortices. ***LinAir*** represents the wing surfaces with discrete vortex lines forming skewed horseshoe vortices. The vortex strengths are adjusted so that the flow is tangent to the surfaces at a series of control points. These points are located on the 3/4 chord line of each element, at the (lateral) center of each panel. The "bound vortex" is located at the 1/4 chord line. The program computes the solution to a linear system of equations:

$$[AIC]\ \{\Gamma_i\} = \{\alpha_i\}$$

where:

[AIC] is a matrix of aerodynamic influence coefficients — the effect of panel i on panel j,

$\{\Gamma_i\}$ is an array of circulation strengths,

$\{\alpha_i\}$ is an array containing the aerodynamic incidence of each panel (its angle of attack with respect to the freestream.

Once this system of equations is solved (using the method of L-U decomposition and Gaussian elimination) the force and moment contribution of each panel is computed from the Kutta-Joukowski relation: $\mathbf{F} = \rho\ \mathbf{V} \times \Gamma$.

Subroutine PLANF

The planform geometry which is consistent with wing sweep, wing dihedral, wing span, joint location, the ratio of wing and tail aspect ratios, and the desired aircraft static margin is determined by the subroutine PLANF. The assumptions are, (1) that the parameters defining the wing geometry are fixed, and (2) that the distance between the aircraft center of gravity and the tail root quarter chord is a fixed input value. The basic idea is to move the wing forward or aft until the desired static margin is obtained. Since the wings are joined this requires changing the tail sweep and dihedral.

The static margin for a given wing position is determined using sm = $-\partial C_M/\partial C_L$, where the moment coefficient is taken about the aircraft center of gravity. In terms of moment and lift coefficients at two discrete angles of attack we have sm = -(CM2 - CM1)/(CL2 -

CL1). However, since $C_M = C_L = 0.0$ for an untwisted planform at zero angle of attack the static margin can be found using sm = -CM2/CL2. The values for CM2 and CL2 are determined in PLANF using the subroutine LASUB with the angle of attack equal to one degree.

The iteration on wing position begins with an initial guess for the total distance between wing and tail root quarter chords (Xtotal). This initial guess is labeled Xtotal1 in Fig. 4, which gives a graphical representation of the determination of wing position (Xtotal). The static margin (sm_1) is calculated for the input value of Xtotal ($Xtotal_1$) and compared with the desired value. The value for Xtotal is then incremented using the following criteria:

$$Xtotal_2 = Xtotal_1 + Xdel$$

where

$$Xdel = (sm_1 - sm_{req})\ Cref$$

The static margin (sm_2) is calculated for $Xtotal_2$, and a linear approximation given by

$$Xtotal = \Big((sm_{req} - sm_1)\ (Xtotal_2 - Xtotal_1) / (sm_2 - sm_1) \Big) + Xtotal_1$$

is used to predict the value of Xtotal which gives $sm = sm_{req}$. This first prediction corresponds to the value of Xtotal at point 3 in Fig. 4. As shown in Fig. 4, the next iteration would include the calculation of the static margin for Xtotal at point 3, thereby obtaining point 4 and predicting Xtotal shown at point 5 using points 2 and 4. If any further iterations are needed the wing position (Xtotal) is predicted using the linear approximation above and the previous two static margin calculations. The convergence criteria is defined by an input error tolerance, where the error is given by $(sm - sm_{req})$.

Once the configuration with the input wing geometry and desired static margin is determined, all the geometric quantities needed for future aerodynamic and structural calculations are determined and returned to JWOPT.

Subroutine TLOAD

The aerodynamic load distributions need to be determined before the structure can be optimized. Subroutine TLOAD is an aerodynamic optimization routine which minimizes the drag, and trims the aircraft at CL = CLref. The aerodynamic solution is determined for an aircraft cruising at zero angle of attack. The drag is minimized with respect to root incidence and tip twist angles for the wing and tail, where the twist distributions are linear. Consequently subroutine TLOAD calculates the lift distributions, root incidence and tip twist angles, and the minimum drag of the lifting system. The basic formulation is described in Appendix A.

Subroutine BEAM

The subroutine BEAM is responsible for the structural analysis. Its primary function is to calculate and return the structural moment distributions, and deflections due to aerodynamic loads, structural weight, and wing joint reactions forces. The structural analysis assumes that the wing and tail can be represented by inextensible beams. These beams are represented by a finite number of sections with known inertial properties. Since moments of inertia are treated externally to the subroutine BEAM, any beam cross-section can be analyzed. The local coordinates about which the moments of inertia are defined are shown in Fig. 5 for a box beam. Variables are passed between the subroutine BEAM and the calling program using a common block in the include file JWOPT.INC.

Structural analysis for joined and conventional wings differs only by the solution for the joint reactions required for the joined wing case. The first step in the solution of the indeterminate joined wing structure is to calculate the beam deflections and rotations due to applied aerodynamic loads, distributed beam weight, and unit joint reaction forces. These deflections and rotations are determined by integrating the second order expressions relating local moments and axial forces to the deflections and rotations. The reaction forces which produce zero relative deflection at the joint are calculated using linear superposition of loads and deflections. This superposition is shown for loads and deflections in the direction of the global Z axis in Fig. 6. We can now represent the joint deflections in global coordinates due to the reaction forces only in terms of the deflections due to the applied loads as;

$$\{\Delta R\} = \{\Delta T_r\} - \{\Delta W_r\} = \{\Delta W_a\} - \{\Delta T_a\}$$

where ΔW and ΔT represent wing and tail deflections respectively and subscripts r and a are used to distinguish deflections due to reaction forces and those due to applied loads. The solution for the joint reactions in global coordinates is now completed using the deflections due to unit joint loads by writing a linear system of equations:

$$[SIC]\{F\} = \{\Delta R\}$$

where $\{F\}$ represents the forces and moments for each coordinate direction, $\{\Delta R\}$ represents the corresponding joint deflections and rotations, and SIC is a matrix of structural influence coefficients. The structural influence coefficients are the joint deflections in global coordinates due to unit reaction loads at the joint. The moment, and shear distributions in local coordinates for each beam are now computed If a conventional "un-joined" configuration is being studied, the subroutine BEAM simply integrates the deflection equations due to the applied loads to determine the deflections, and calculates the shear and moment distributions with the reaction forces equal to zero.

Moments of Inertia and Beam Dimension Optimization

Box beams are used for both the wing and tail and are optimized for minimum total structural weight. The cross sections of these beams are shown in Fig. 5. The web and cap thicknesses are sized such that a maximum allowable stress is not exceeded and minimum cross sectional area is obtained at each spanwise station. A linearized form of the beam moments of inertia is used so that the cap and web thicknesses, tz and tx respectively, can be related by a cubic function. Since the roots of a cubic equation are given in closed form, a one variable optimizer is used quite efficiently to determine tz, and tx. The linearized inertias are given by;

allowable stress constraint becomes this equation *

* JWAOPT uses closed form expression for A1, Az, Tx, Tz (see AIAA-90-3197)

$$Iz = 4\, Cz\, Cx^2\, tx + (4/3)\, Cx^3\, tz$$

$$Ix = 4\, Cx\, Cz^2\, tz + (4/3)\, Cz^3\, tx$$

$$J = 4\, Cx^2\, Cz^2\, tx\, tz \,/\, (Cx\, tx + Cz\, tz)$$

where the torsional moment of inertia is derived using Brett's formula[8]. The maximum normal stress at each spanwise station is given by:

$$\sigma_{max} = |Mz|\, Cx/Iz + |Mx|\, Cz/Ix + |Fy| /A$$

The values of t_x and t_z which result in $\sigma < \sigma_{max}$ with minimum weight are determined from these expressions. (Note that shear loads have been excluded here. A future version of the code will consider these stresses as well.) the expressions above are used to determine the cubic function which relates tx, and tz. The one variable optimizer uses beam cross sectional area as the objective function to calculate tx and tz which give minimum beam weight. The structural weight is obtained by multiplying the material density by the beam volume, which is calculated by integrating the cross sectional area distribution along the span numerically.

Output structural weight is for semi-span only.

Input, Output, and Sample Case

JWOPT is executed in batch mode with the input file name specified interactively. Once the input file name is given, the program runs automatically to completion. A complete list of all program input variables are shown in Table 1 with a brief description of each variable. All of these variables are read from the input file using list directed (free-format) i/o. The input file for the sample case is shown in Fig. 7. All of the lines in the file JWOPT.DAT which have an exclamation point in the first column are considered comment lines and are ignored. As shown in Fig. 7 this enables the user to place variable names and or physical descriptions in the data file.

The input parameters btbw and Npan(1) are used to calculate the number of panels for the tail using Npan(2) = ifix(btbw*Npan(1)). Hence btbw and Npan(1) should be selected such that Npan(2) is an integer. The format for the input file need not be changed when switching from conventional to joined wing configurations. However, when running a joined wing case the tail sweep and dihedral angles are calculated from other geometric parameters to guarantee the proper joint location. Consequently, any tail sweep and dihedral angle can be specified when analyzing a joined wing. The multiple case option parameter nparam is set equal to zero for a one case only run. The values input for Dparam and ncase are ignored when nparam is equal to zero. When running the multiple case option, nparam specifies the parameter which is varied, Dparam is the value used to increment the chosen parameter, and ncase specifies the maximum number of cases. For example, nparam=1, Dparam=5.0, and ncase=5 will optimize the input configuration for five different values of wing sweep beginning with the starting value. If sweep(1) is set equal to 20 degrees the five joined wing optimization cases correspond to wing sweep angles of 20, 25, 30, 35, and 40 degrees.

The results are written for each case to any or all of seven output files. These output files are selected using input array nout. As shown in Table 1 if an array element in nout is set equal to 1 the output quantities corresponding to that array location are printed. Otherwise, printing is suppressed. The seven output choices are written to the seven output files given by;

AERO.OUT	The aerodynamic force and coefficient distributions which give minimum induced drag.
ALOAD.OUT	The applied load distributions in the local z and x directions.
SLOAD.OUT	The shear and moment distributions of the wing and tail in local coordinates.
DEFLECT.OUT	The deflections and coordinates of the deflected shape of the quarter chord lines in global coordinates. The joint reactions in both local and global coordinates.
THICK.OUT	The thickness and weight distributions of the beam caps and webs.
WEIGHT.OUT	The convergence history of the wing, tail, and total structural weight (lb.). {Half of aircraft or semi-span}
WING.XYZ	The three dimensional coordinates of the wing and

tail, including the quarter chord lines.

where the three character extension 'OUT' is specified interactively by the user.

The case number is appended to the output file name; for example AERO.OUT1 is the first output file for case number one. When the multiple case option is used the output files DvsP.OUT and WvsP.OUT are created. These output files contain the drag coefficients and the structural weights for each value of the selected parameter. A detailed description of the program output variables is given in Table 2. Examples of each of the seven output files, created by running JWOPT with the sample input file shown in Fig. 7, are shown in Fig. 8 through Fig. 14. Examples of output files DvsP.OUT and WvsP.OUT are shown in figures 15 and 16 respectively. All of these output files are compatible with the plotting program QuickPlot version 1.2 for the Apple Macintosh computers by Desktop Engineering[9]. Two example plots are shown in Fig. 17 and 18, which display the applied load in the local z-direction and the structural moment distribution about the local x-axis. The data for these plots is found on files ALOAD.OUT1 and SLOAD.OUT1 as shown in Fig. 9 and 10 respectively.

References

1. Wolkovitch, J.: "The Joined Wing: An Overview." *Journal of Aircraft*, March 1986, pp.161-178.

2. Wolkovitch, J.: "Joined-Wing Research Airplane Feasibility Study." AIAA paper 84-2471, Presented at the AIAA/AHS/ASEE Aircraft Design Systems and Operations Meeting, San Diego, California, Oct. 31 - Nov. 2, 1984.

3. White, E. R.: "Preliminary Force Test Data for a Joined Wing Aircraft Configuration." NASA Langley Research Center, Hampton, Virginia, Internal Memo File V-19100/OLTR-094, April 8, 1980.

4. Hajela, P.: "Weight Evaluation of the Joined Wing Configuration." Final Report, Department of Engineering Sciences, University of Florida, Gainesville, Florida, 1983.

5. Miura, H.; Shyu, A.; and Wolkovitch, J.: " Parametric Weight Evaluation of Joined Wings by Structural Optimization." AIAA Paper 85-0642-CP, April 1985.

6. Samuels, M. F.: " Structural Weight Comparison of a Joined Wing and a Conventional Wing." *Journal of Aircraft*, Vol. 19, June 1982, pp. 485-491.

7 Kroo, I., *LinAir User's Guide*, Desktop Aeronautics, P.O. Box 9937, Stanford CA, 94305, Aug. 1987.

8 Timoshenko, Young, *Elements of Strength of Materials*, van Nostrand, 1968

9. *QuickPlot User's Guide*, Desktop Engineering, P.O. Box 2401, Stanford CA, 94305, March 1987

Table 1. Input variables and descriptions

Program Variables	Description of Input Variables
E	Young's Modulus (psi.)
G	Shear Modulus (psi.)
sigma	~~Ultimate~~ yield stress (psi.)
safety	Safety factor (usually about 1.5)
loadf	Load factor (N_{Limit})
gweight	Aircraft weight (lbs.)
density	Material density (lbs./in.^{3})
tstart	Initial thickness (in.) for the beam webs and caps.
tmin	Minimum gauge for beam web and cap thicknesses (in.). % c
toverc	Average streamwise thickness to chord ratio for the wing and the tail.
StrBoxC	Ratio of structural beam chord to wing and or tail chord. 
WeightTol	The weight tolerance used for convergence to minimum structural weight.
NSTOP	The maximum number of allowed for convergence to minimum structural weight.
hvert	Height of vertical tail, or the vertical gap between the wing and tail root sections in (ft.).
taper(i)	Taper ratio (tip chord/root chord) of the wing and tail. i=1 wing, 2 -tail
ARR	The ratio of tail aspect ratio to wing aspect ratio where the areas and spans are measured in the plan view.
btbw	The ratio of tail span to wing span, where the spans are measured in the plan view.
dihed(i)	Wing and tail dihedral angles (degrees) measured in the front view
sweep(i)	Wing and tail sweep angles (degrees) measured in the plane of the wing or tail quarter chord line respectively.
jtype (Joined)	Flag which indicates joined or conventional configuration. If j is set equal to one a joined wing configuration is considered, otherwise a conventional structure is studied.
ism (FixWing)	Flag which indicates a fixed wing position and input static margin, or a wing position which is to be determined such that the desired static margin is obtained.
bref	The reference wing ~~area~~ span. This should be the aircraft wing ~~area~~ span.
Sref	The reference area. This should be equal to the wing area plus the tail area,

i=2 not used for Joined-Wing Case. Sweep and dihed of tail calculat

	where the areas are measured in the plane of the wing and tail respectively.
CLref	The required airplane cruise lift coefficient based on Sref. $W / q\,S_{ref}$
Mach	Free steam Mach number. (0.0 → ± 0.6)
Npan(1)	The number of wing panels used for both aerodynamic and structural analysis. (for semi-span)
Xtotal	The distance in the global x-direction between the wing root quarter chord and the tail root quarter chord (ft.).
Xtail	The distance in the global x-direction between the aircraft center of gravity and the tail root quarter chord (ft.).
smTOL	The error tolerance between the actual and required static margin. This is a difference in static margins.
smreq	The required static margin: $\partial Cm/\partial CL$ with Cm based on Cref = Sref/bref
nparam	Flag which indicates which parameter is to be varied in the multiple case option. 0 = Run input case only. 1 = sweep(1). 2 = btbw. 3 = ARR. 4 = dihed(1).
Dparam	The increment in the chosen parameter when running the multiple case option.
ncase	The maximum number of cases to be optimized.
CDc(i)	Wing and tail constant parasite drag coefficients.
CDL(i)	Wing and tail linear parasite drag terms.
CDq(i)	Wing and tail quadratic parasite drag terms.
nout(i)	Output selection array. If nout(i) is set equal to one print selected output, otherwise suppress printing. Output selections are: i=1 Wing and tail coordinates in 3D in WING.XYZ. i=2 Applied loads in ALOAD.ext i=3 Deflections and joint reactions in DEFLECT.ext. i=4 Shear and Moment distributions in SLOAD.ext. i=5 Convergence of structural weight in WEIGHT.ext i=6 Beam web and cap thickness and weight distributions in THICK.ext. i=7 Aerodynamic force and coefficient distributions which give minimum induced drag in AERO.ext. The three letter extension 'ext' is specified by the user interactively.

Cm_0 – Zero lift pitching moment caused by everything except wing and tail twist.

Table 2. Output variables and descriptions

Variable	Description
xWRQC	X coordinate of wing root quarter chord.
yWRQC	Y coordinate of wing root quarter chord.
zWRQC	Z coordinate of wing root quarter chord.
xTTQC	X coordinate of tail tip quarter chord.
yTTQC	Y coordinate of tail tip quarter chord.
zTTQC	Z coordinate of tail tip quarter chord.
xWRLE	X coordinate of wing root leading edge.
yWRLE	Y coordinate of wing root leading edge.
zWRLE	Z coordinate of wing root leading edge.
xWTLE	X coordinate of wing tip leading edge.
yWTLE	Y coordinate of wing tip leading edge.
zWTLE	Z coordinate of wing tip leading edge.
xWTTE	X coordinate of wing tip trailing edge.
yWTTE	Y coordinate of wing tip trailing edge.
zWTTE	Z coordinate of wing tip trailing edge.
xWRTE	X coordinate of wing root trailing edge.
yWRTE	Y coordinate of wing root trailing edge.
zWRTE	Z coordinate of wing root trailing edge.
xTRLE	X coordinate of tail root leading edge.
yTRLE	Y coordinate of tail root leading edge.
zTRLE	Z coordinate of tail root leading edge.
xTTLE	X coordinate of tail tip leading edge.
yTTLE	Y coordinate of tail tip leading edge.
zTTLE	Z coordinate of tail tip leading edge.
xTTTE	X coordinate of tail tip trailing edge.
yTTTE	Y coordinate of tail tip trailing edge.
zTTTE	Z coordinate of tail tip trailing edge.
xTRTE	X coordinate of tail root trailing edge.
yTRTE	Y coordinate of tail root trailing edge.
zTRTE	Z coordinate of tail root trailing edge.
xWTQC	X coordinate of wing tip quarter chord.

yWTQC	Y coordinate of wing tip quarter chord.
zWTQC	Z coordinate of wing tip quarter chord.
Y(i,j)	Location of the wing and tail panel center points measured from the wing and tail root quarter chords respectively, in the local y directions.
Pz(i,j)	Applied load including the weight distribution for the optimized structure in the local z-direction (lb./in).
Px(i,j)	Applied load distribution in the local x-direction (lb./in.)
D(1), Dlocal(1)	Joint reaction force in x-direction. Given in both global and local coordinates.
D(2), Dlocal(2)	Joint reaction force in y-direction. Given in both global and local coordinates.
D(3), Dlocal(3)	Joint reaction force in z-direction. Given in both global and local coordinates.
D(4), Dlocal(4)	Joint reaction moment about the x-axis. Given for both global and local coordinates.
D(5), Dlocal(5)	Joint reaction moment about the y-axis. Given for both global and local coordinates.
D(6), Dlocal(6)	Joint reaction moment about the z-axis. Given for both global and local coordinates.
X1	X coordinate of the deflected wing or tail quarter chord in the global system.
Y1	Y coordinate of the deflected wing or tail quarter chord in the global system.
Z1	Z coordinate of the deflected wing or tail quarter chord in the global system.
DX1	Deflection in the global x-direction.
DY1	Deflection in the global y-direction.
DZ1	Deflection in the global z-direction.
Fxbeam(i,j)	Shear distribution for the wing and tail in the local x-direction.
Fybeam(i,j)	Axial load distribution for the wing and tail in the local y-direction.
Fzbeam(i,j)	Shear distribution for the wing and tail in the local z-direction.
Mnx(i,j)	Moment distribution for the wing and tail about the local x-axis.
Mny(i,j)	Moment distribution for the wing and tail about the local y-axis.
Mnz(i,j)	Moment distribution for the wing and tail about the local z-axis.

ncnvrg	Iteration number for structural weight optimization loop in JWOPT.
BeamWt(ncnvrg,1)	Weight optimization history for the wing (lbs.).
BeamWt(ncnvrg,2)	Weight optimization history for the tail (lbs.).
tweb(i,j)	Thickness distribution of the beam web for the wing and tail (in.).
tcap(i,j)	Thickness distribution of the beam cap for the wing and tail (in.).
Wweb	Weight distribution of the beam web for the wing and tail (lb./in.).
Wcap	Weight distribution of the beam cap for the wing and tail (lb./in).
CLsec(i,j)	Section lift coefficient distribution.
SecL(i,j)	Section lift distribution.
chord(i,j)	Section chord in inches.
CD	Drag coefficient of the lifting system

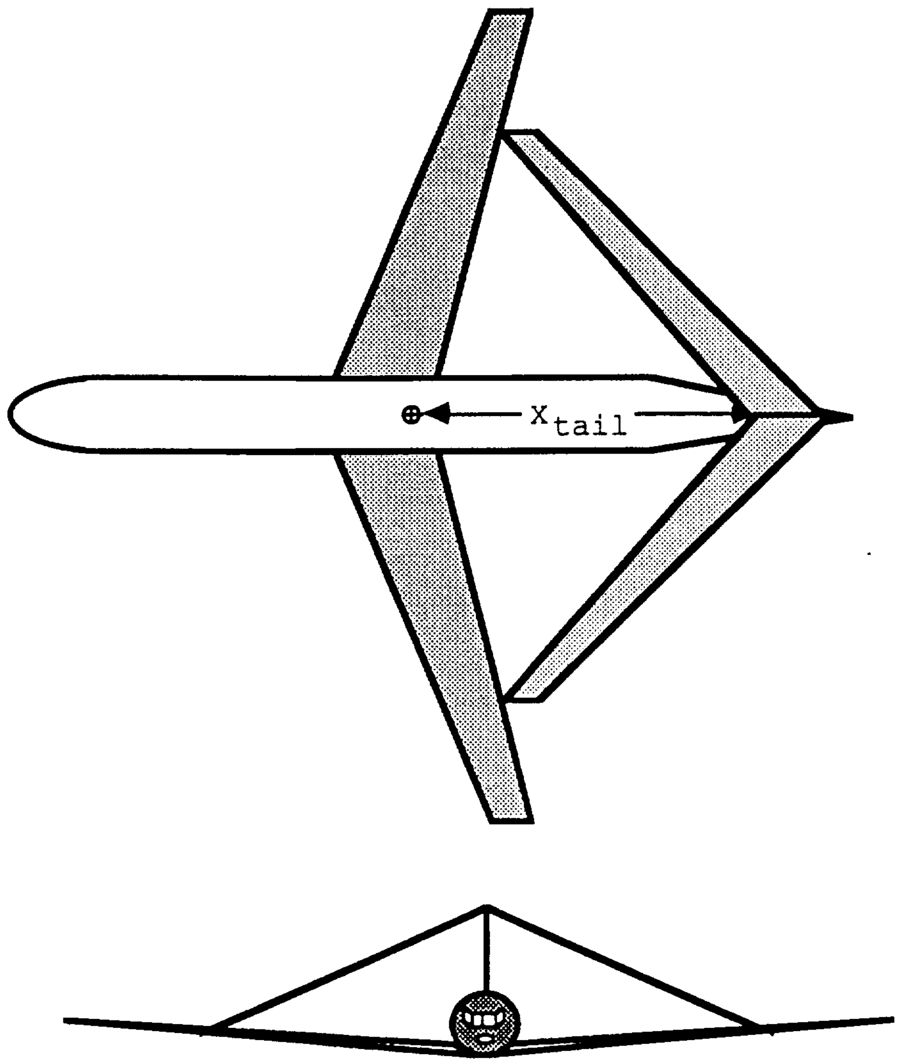

Figure 1. Joined Wing Basic Geometry

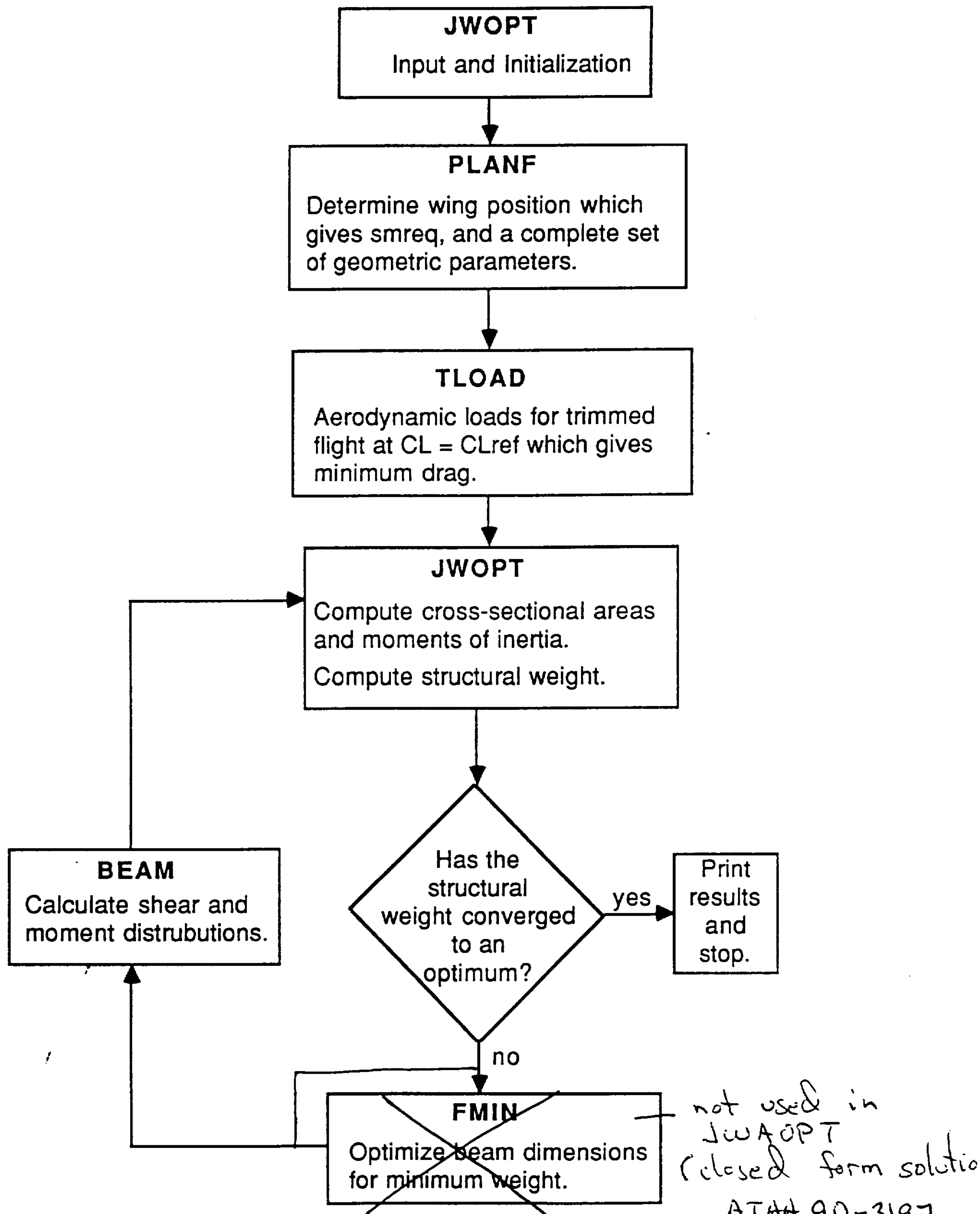

Fig. 2. Flowchart displaying the sequence of operations in program JWOPT.

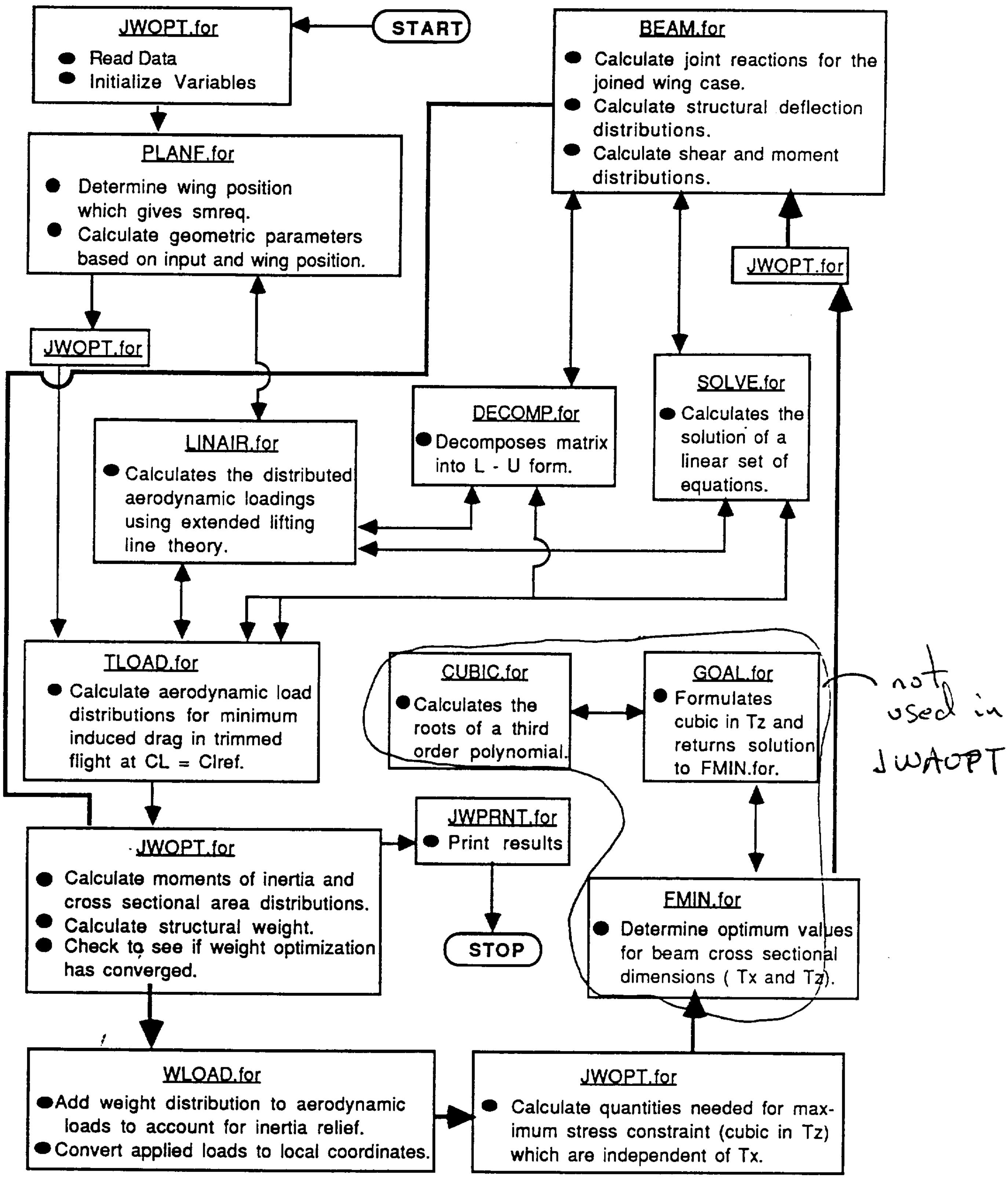

Fig. 3. A detailed flowchart displaying operations within JWOPT.

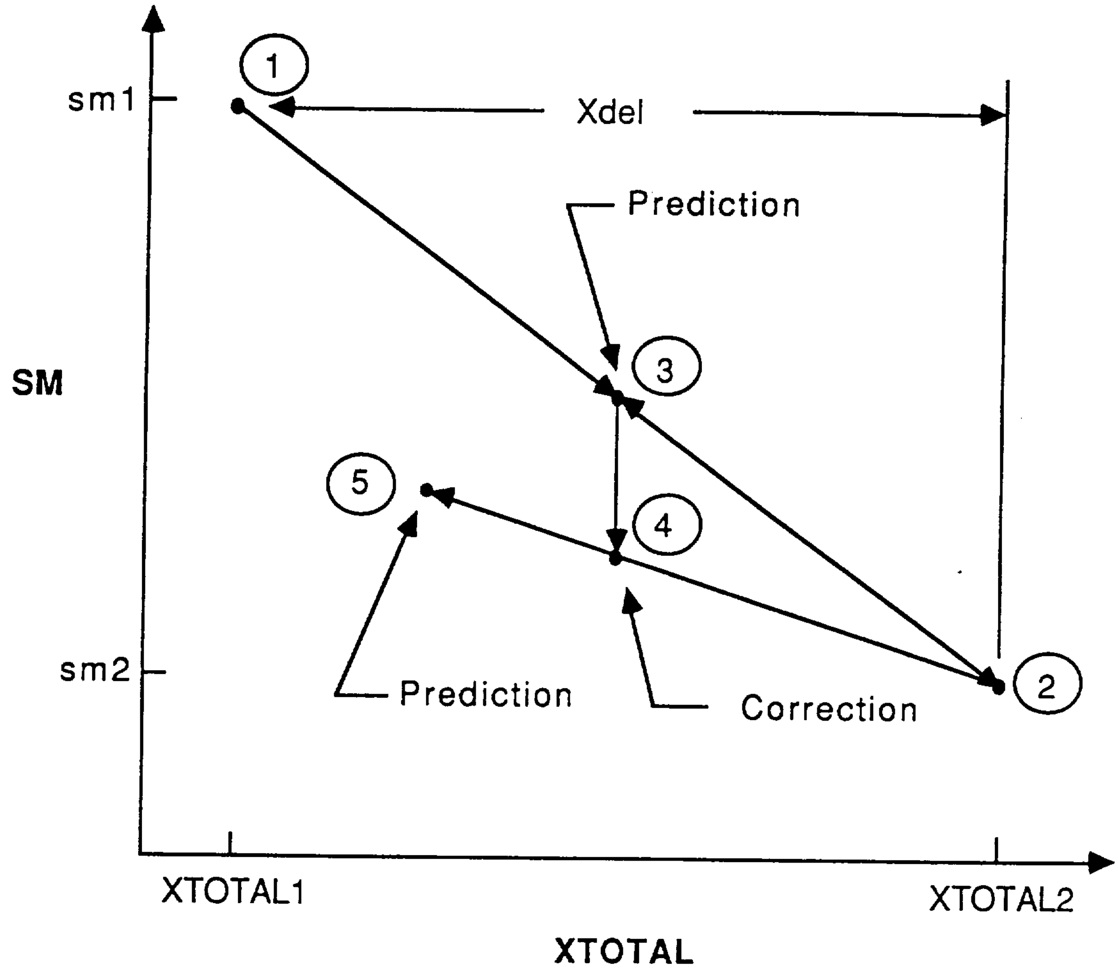

Fig. 4. Iteration on wing position for desired static margin.

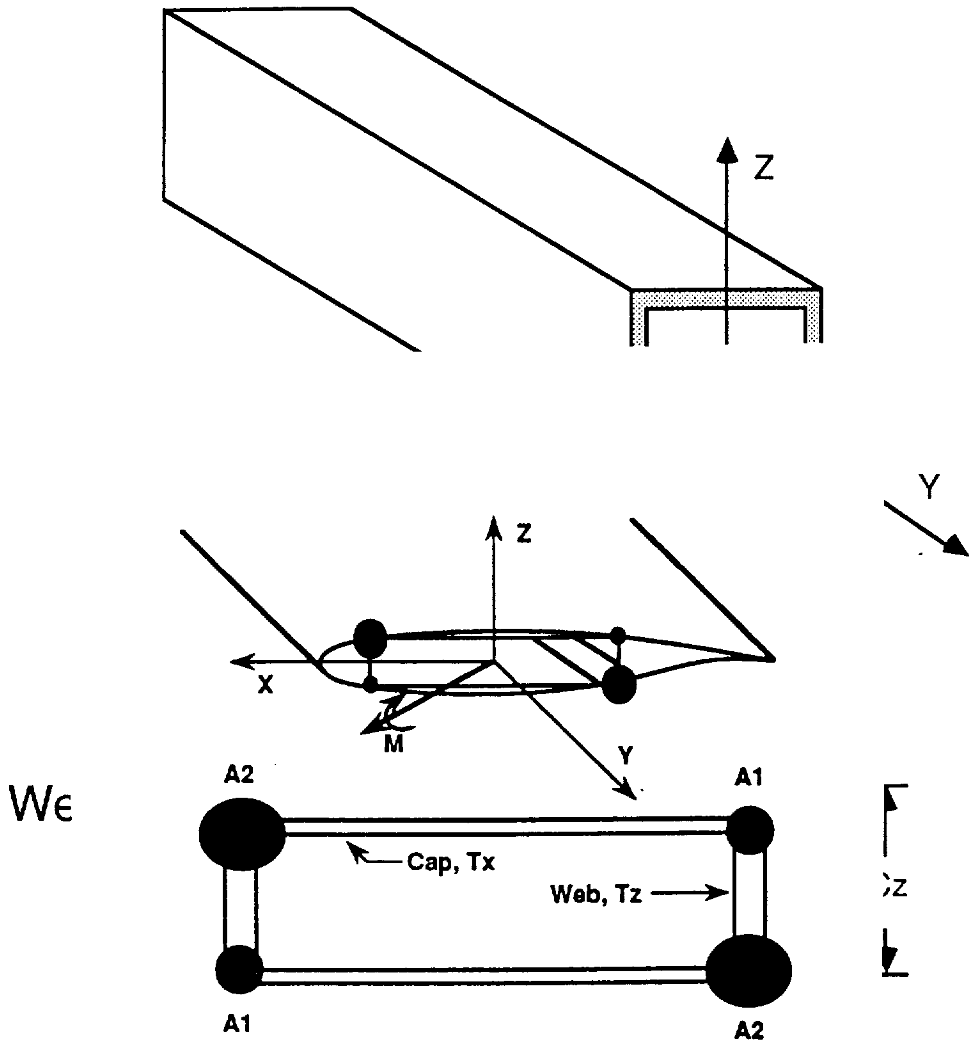

Figure 3: Structural box model with skins, stringers, and the resultant bending moment.

Fig. 5. Box beam in local coordinate system.

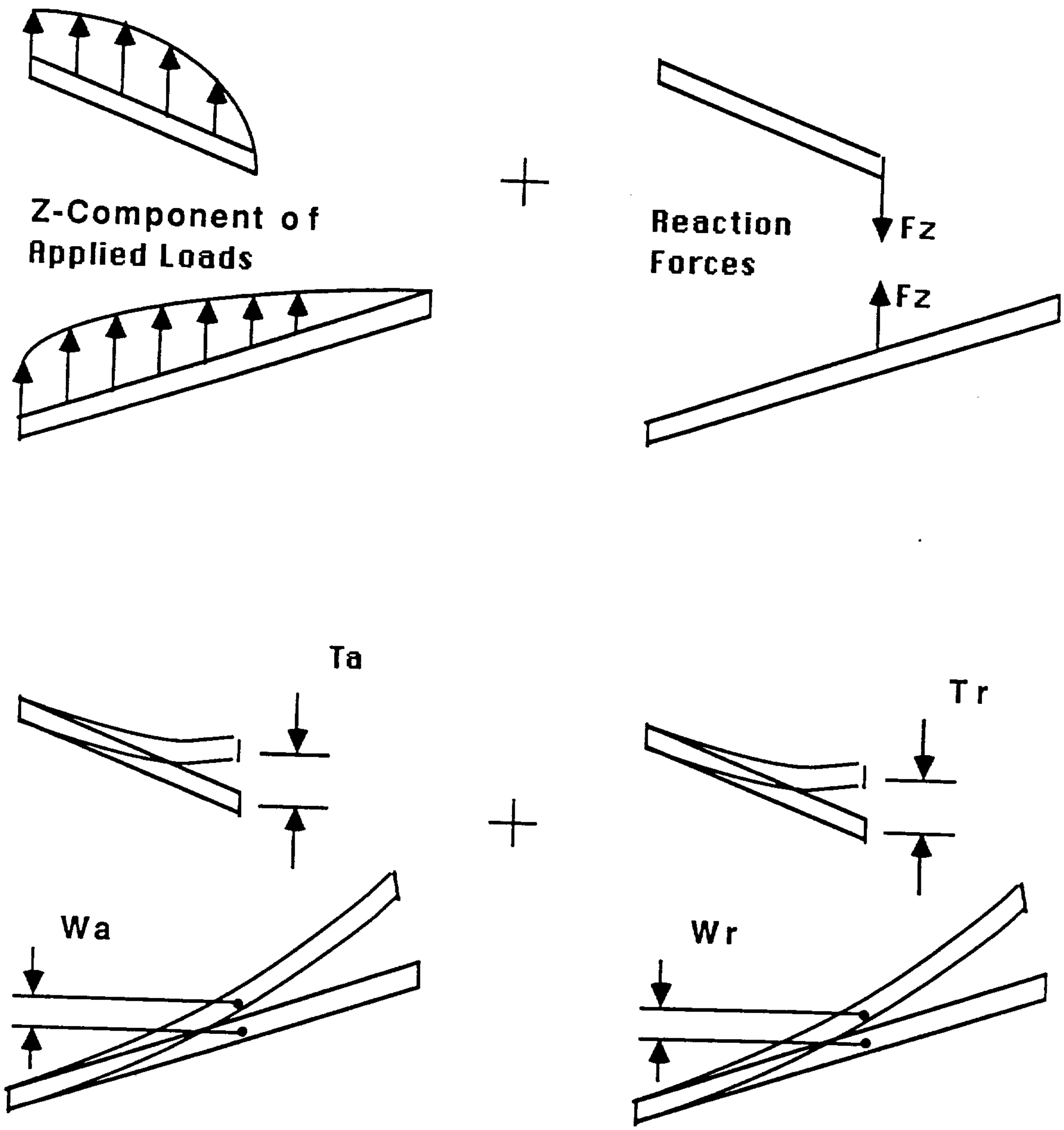

Fig. 6. Superposition of loads and deflections in the global Z-direction.

```
!  Input file for JWOPT.FOR
!  (Note that lines beginning with ! are comments.)
!  Structural properties
!  ----------------------
!  Eyoung   G         sigma      safety   loadf    gweight   Density
!  (psi)    (psi)     (psi)                        (lbs)     (lb/cu.in)
   1.00e+07 3.70e+06  3.70e+04   1.5        2.5    100000.0    .1
!
!  Beam geometry        dstart = √A1 = √A2
!  --------------
!  tstart  tmin    toverc   StrBoxC   WeightTol (lbs.)   NSTOP
   0.60    0.040   0.12     0.60      5.0                30
!
!  Wing and tail geometry
!  ----------------------
!  hvert (ft.)   taper(1)   taper(2)   ARR       btbw
   20.0205       0.25       0.40       1.0       0.6
!
!  dihed(1)  sweep(1)  dihed(2)  sweep(2)  Joined (1=yes)  FixWing (1=yes)
   5.00      20.00     -20.00    -20.00    1               0
!
!  Refference Parameters
!  ------------------
!  bref     Sref          CLref     Mach    Npan(1)
!  (ft.)    (sq. ft.)
   93.3     1268.0        0.3164    0.5     20
!
!  Center of gravity location and static margin parameters
!  -------------------------------------------------------
!  Xtotal   Xtail    smTOL    smreq      CM0
!  (ft.)    (ft.)
   32.00    35.0     0.001    0.5900     -.0359
!
!  Multiple case option parameter
!  ------------------------------
!  nparam        Dparam        ncase
   1             5.0           5
!
!  Wing and tail parasite drag factors:
!  ------------------------------------
!  CDc(1)   CDL(1)   CDq(1)
   0.007    0        0.003
!  CDc(2)   CDL(2)   CDq(2)
   0.007    0        0.003
!
!  Possible output options (1=write to file):
!  ------------------------------------------
! 3-D       Aero       Deflections   Moments     Weight       Material     Aerodynamic
! Geom      Loads      & Joint RxN   & shear     History      thickness    forces
! WING.xyz  ALOAD.OUT  DEFLECT.out   SLOAD.out   WEIGHT.out   THICK.out    AERO.out
   1         1          1             1           1            1            1
end
```

Fig. 7. Sample case input file for JWOPT.

```
! RESULTS FROM TLOAD
! CL =  .3164 CD =  1.2560E-02 CM =  3.5900E-02
! CL1(2) =  -6.8422E-02 CL1(1) =  .3848 Lbar =  -.1778
! Incidence and twist =  7.091 -2.795 -.8395 -.7547
TITLE:Aerodynamic Forces
SUBTITLE: M =  .5000 CL =  .3164
XLABEL:Distance along unswept bound vortex (in.)
YLABEL:Clsec, SecL, chord
head:y Clsec SecL chord
Legend:wing
  0.14048E+02  0.49958E+00  0.54892E+00  0.17919E+03
  0.42145E+02  0.51694E+00  0.54629E+00  0.17235E+03
  0.70242E+02  0.53197E+00  0.53984E+00  0.16550E+03
  0.98339E+02  0.54464E+00  0.52982E+00  0.15865E+03
  0.12644E+03  0.55511E+00  0.51669E+00  0.15180E+03
  0.15453E+03  0.56357E+00  0.50091E+00  0.14495E+03
  0.18263E+03  0.57020E+00  0.48285E+00  0.13810E+03
  0.21073E+03  0.57509E+00  0.46285E+00  0.13126E+03
  0.23882E+03  0.57835E+00  0.44119E+00  0.12441E+03
  0.26692E+03  0.58001E+00  0.41810E+00  0.11756E+03
  0.29502E+03  0.58012E+00  0.39382E+00  0.11071E+03
  0.32311E+03  0.57882E+00  0.36863E+00  0.10386E+03
  0.35121E+03  0.57662E+00  0.34301E+00  0.97016E+02
  0.37931E+03  0.57397E+00  0.31734E+00  0.90167E+02
  0.40741E+03  0.57012E+00  0.29127E+00  0.83319E+02
  0.43550E+03  0.56375E+00  0.26434E+00  0.76471E+02
  0.46360E+03  0.55290E+00  0.23604E+00  0.69623E+02
  0.49170E+03  0.53338E+00  0.20531E+00  0.62775E+02
  0.51979E+03  0.49413E+00  0.16945E+00  0.55927E+02
  0.54789E+03  0.39728E+00  0.11956E+00  0.49078E+02
end
Legend:tail
  0.16524E+02 -0.27043E+00 -0.15817E+00  0.95385E+02
  0.49573E+02 -0.26168E+00 -0.14520E+00  0.90494E+02
  0.82621E+02 -0.25612E+00 -0.13443E+00  0.85602E+02
  0.11567E+03 -0.25751E+00 -0.12744E+00  0.80710E+02
  0.14872E+03 -0.26342E+00 -0.12246E+00  0.75819E+02
  0.18177E+03 -0.27175E+00 -0.11818E+00  0.70927E+02
  0.21482E+03 -0.28098E+00 -0.11377E+00  0.66036E+02
  0.24786E+03 -0.28984E+00 -0.10867E+00  0.61144E+02
  0.28091E+03 -0.29684E+00 -0.10239E+00  0.56253E+02
  0.31396E+03 -0.29925E+00 -0.94243E-01  0.51361E+02
  0.34701E+03 -0.29003E+00 -0.82641E-01  0.46470E+02
  0.38006E+03 -0.24203E+00 -0.61703E-01  0.41578E+02
end
```

Fig. 8. Output file AERO.OUT1 for sample case.

```
TITLE:Applied Load Distributions
SUBTITLE: M =  .5000 CL =  .3164
XLABEL:Distance From Wing Or Tail Root (in.)
YLABEL:pz(i,j),px(i,j),py(i,j) lb/in
! Wing And Tail Applied Load Distributions in local coordinates.
head: Y             Pz            Px            Py
legend:wing
   0.14950E+02   0.34438E+03   0.13444E+02   0.29917E+00
   0.44850E+02   0.34577E+03   0.13288E+02   0.49579E+00
   0.74750E+02   0.34451E+03   0.12717E+02   0.67977E+00
   0.10465E+03   0.34067E+03   0.11904E+02   0.84244E+00
   0.13455E+03   0.33411E+03   0.10964E+02   0.94888E+00
   0.16445E+03   0.32500E+03   0.99570E+01   0.98866E+00
   0.19435E+03   0.31365E+03   0.89107E+01   0.96646E+00
   0.22425E+03   0.30114E+03   0.78105E+01   0.95215E+00
   0.25415E+03   0.28641E+03   0.66812E+01   0.84357E+00
   0.28405E+03   0.27024E+03   0.54501E+01   0.68853E+00
   0.31395E+03   0.25370E+03   0.38922E+01   0.55786E+00
   0.34385E+03   0.23719E+03   0.91727E-01   0.50509E+00
   0.37375E+03   0.22150E+03   0.13403E+02   0.56805E+00
   0.40365E+03   0.20581E+03   0.76947E+01   0.60942E+00
   0.43355E+03   0.18974E+03   0.60251E+01   0.63670E+00
   0.46345E+03   0.17296E+03   0.50282E+01   0.64871E+00
   0.49335E+03   0.15508E+03   0.43262E+01   0.64410E+00
   0.52325E+03   0.13534E+03   0.38298E+01   0.61622E+00
   0.55315E+03   0.11164E+03   0.35068E+01   0.53960E+00
   0.58305E+03   0.78639E+02   0.31481E+01   0.45588E+00
end
legend:tail
   0.21431E+02  -0.91780E+02   0.41669E+01   0.12313E+01
   0.64294E+02  -0.83400E+02   0.27088E+01   0.64530E+00
   0.10716E+03  -0.76246E+02   0.17834E+01   0.10425E+00
   0.15002E+03  -0.71494E+02   0.12660E+01  -0.26172E+00
   0.19288E+03  -0.69838E+02   0.16121E+01   0.34264E+00
   0.23575E+03  -0.68154E+02   0.18136E+01   0.75436E+00
   0.27861E+03  -0.66088E+02   0.19032E+01   0.10197E+01
   0.32147E+03  -0.63308E+02   0.18753E+01   0.11231E+01
   0.36433E+03  -0.59483E+02   0.17152E+01   0.10305E+01
   0.40720E+03  -0.54118E+02   0.13928E+01   0.68078E+00
   0.45006E+03  -0.47146E+02   0.12597E+01   0.45614E+00
   0.49292E+03  -0.38116E+02   0.24262E+01   0.16330E+01
end
```

Fig. 9. Output file ALOAD.OUT1 for sample case.

```
TITLE:Shear and Moment Distributions
SUBTITLE:
XLABEL:Dist. from wing or tail root (in.)
YLABEL:Mx,My,Mz (lb.-in.), Fx,Fy,Fz (lb.)
! Shear and Moment Distributions
head: Y       Fx      Fy      Fz      Mx      My      Mz
legend:wing
  0.14950E+02 -0.12418E+06  0.52354E+05  0.57674E+05  0.47521E+07  0.42711E+06  0.42583E+08
  0.44850E+02 -0.12457E+06  0.52354E+05  0.47356E+05  0.31816E+07  0.42711E+06  0.38864E+08
  0.74750E+02 -0.12496E+06  0.52354E+05  0.37036E+05  0.19202E+07  0.42711E+06  0.35133E+08
  0.10465E+03 -0.12533E+06  0.52355E+05  0.26793E+05  0.96679E+06  0.42711E+06  0.31391E+08
  0.13455E+03 -0.12567E+06  0.52355E+05  0.16705E+05  0.31797E+06  0.42711E+06  0.27638E+08
  0.16445E+03 -0.12599E+06  0.52355E+05  0.68509E+04 -0.32146E+05  0.42711E+06  0.23876E+08
  0.19435E+03 -0.12627E+06  0.52355E+05 -0.26970E+04 -0.91711E+05  0.42711E+06  0.20104E+08
  0.22425E+03 -0.12652E+06  0.52355E+05 -0.11888E+05  0.12913E+06  0.42711E+06  0.16325E+08
  0.25415E+03 -0.12673E+06  0.52355E+05 -0.20672E+05  0.61920E+06  0.42711E+06  0.12538E+08
  0.28405E+03 -0.12692E+06  0.52354E+05 -0.28994E+05  0.13653E+07  0.42711E+06  0.87460E+07
  0.31395E+03 -0.12706E+06  0.52354E+05 -0.36827E+05  0.23531E+07  0.42711E+06  0.49487E+07
  0.34385E+03 -0.12712E+06  0.52354E+05 -0.44166E+05  0.35676E+07  0.42711E+06  0.11480E+07
  0.35880E+03 -0.12712E+06  0.52354E+05 -0.47712E+05  0.42809E+07  0.42711E+06 -0.75241E+06
  0.35880E+03  0.14042E+04  0.00000E+00  0.37995E+05  0.38057E+07  0.00000E+00 -0.12323E+06
  0.37375E+03  0.12038E+04  0.56805E+00  0.34683E+05  0.32377E+07  0.00000E+00 -0.10224E+06
  0.40365E+03  0.88838E+03  0.60942E+00  0.28295E+05  0.22997E+07  0.00000E+00 -0.72238E+05
  0.43355E+03  0.68327E+03  0.63670E+00  0.22381E+05  0.15457E+07  0.00000E+00 -0.49115E+05
  0.46345E+03  0.51802E+03  0.64871E+00  0.16959E+05  0.96129E+06  0.00000E+00 -0.31379E+05
  0.49335E+03  0.37817E+03  0.64410E+00  0.12055E+05  0.53153E+06  0.00000E+00 -0.18138E+05
  0.52325E+03  0.25624E+03  0.61622E+00  0.77128E+04  0.24042E+06  0.00000E+00 -0.87641E+04
  0.55315E+03  0.14656E+03  0.53960E+00  0.40204E+04  0.70305E+05  0.00000E+00 -0.28145E+04
  0.58305E+03  0.47065E+02  0.45588E+00  0.11757E+04  0.00000E+00  0.00000E+00  0.00000E+00
end
legend:tail
  0.21431E+02  0.10635E+02 -0.16265E+06 -0.19736E+05 -0.17591E+07 -0.62260E+05  0.60121E+06
  0.64294E+02 -0.13672E+03 -0.16265E+06 -0.15981E+05 -0.99748E+06 -0.62260E+05  0.59784E+06
  0.10716E+03 -0.23300E+03 -0.16265E+06 -0.12560E+05 -0.38908E+06 -0.62260E+05  0.58949E+06
  0.15002E+03 -0.29835E+03 -0.16265E+06 -0.93937E+04  0.79231E+05 -0.62260E+05  0.57787E+06
  0.19288E+03 -0.36003E+03 -0.16265E+06 -0.63647E+04  0.41620E+06 -0.62260E+05  0.56392E+06
  0.23575E+03 -0.43345E+03 -0.16265E+06 -0.34074E+04  0.62485E+06 -0.62260E+05  0.54700E+06
  0.27861E+03 -0.51311E+03 -0.16265E+06 -0.53039E+03  0.70830E+06 -0.62260E+05  0.52676E+06
  0.32147E+03 -0.59408E+03 -0.16265E+06  0.22427E+04  0.67032E+06 -0.62260E+05  0.50302E+06
  0.36433E+03 -0.67103E+03 -0.16265E+06  0.48743E+04  0.51604E+06 -0.62260E+05  0.47583E+06
  0.40720E+03 -0.73764E+03 -0.16265E+06  0.73089E+04  0.25247E+06 -0.62260E+05  0.44549E+06
  0.45006E+03 -0.79449E+03 -0.16265E+06  0.94791E+04 -0.11053E+06 -0.62260E+05  0.41260E+06
  0.49292E+03 -0.87348E+03 -0.16265E+06  0.11306E+05 -0.56014E+06 -0.62260E+05  0.37738E+06
end
```

Fig. 11. Output file DEFLECT.OUT1 for sample case.

```
! TITLE:BEAM DEFLECTIONS AND JOINT REACTIONS
! JOINT REACTIONS
!       GLOBAL          LOCAL
! i     D(i)            Dlocal(i)
! 1    -0.10286E+06    -0.12852E+06
! 2     0.10027E+06     0.52354E+05
! 3    -0.77261E+05    -0.85706E+05
! 4     0.59257E+06     0.47514E+06
! 5     0.29278E+06     0.42711E+06
! 6    -0.60597E+06    -0.62918E+06
! WING COORDINATES & DEFLECTIONS
!   X1           Y1           Z1           DX1           DY1           DZ1
  0.51103E+01  0.13995E+02  0.12359E+01 -0.28931E-02  0.50753E-04  0.11502E-01
  0.15313E+02  0.41986E+02  0.37750E+01 -0.26805E-01  0.89021E-03  0.10177E+00
  0.25489E+02  0.69979E+02  0.63991E+01 -0.76757E-01  0.38006E-02  0.27710E+00
  0.35636E+02  0.97976E+02  0.90976E+01 -0.15621E+00  0.10988E-01  0.52676E+00
  0.45749E+02  0.12598E+03  0.11850E+02 -0.27049E+00  0.26186E-01  0.83027E+00
  0.55819E+02  0.15400E+03  0.14628E+02 -0.42605E+00  0.54175E-01  0.11600E+01
  0.65844E+02  0.18203E+03  0.17410E+02 -0.62774E+00  0.98725E-01  0.14931E+01
  0.75821E+02  0.21009E+03  0.20192E+02 -0.87746E+00  0.16082E+00  0.18262E+01
  0.85752E+02  0.23815E+03  0.23004E+02 -0.11731E+01  0.23708E+00  0.21891E+01
  0.95643E+02  0.26623E+03  0.25902E+02 -0.15079E+01  0.32008E+00  0.26387E+01
  0.10550E+03  0.29430E+03  0.28949E+02 -0.18741E+01  0.40159E+00  0.32362E+01
  0.11534E+03  0.32236E+03  0.32213E+02 -0.22628E+01  0.47222E+00  0.40520E+01
  0.12517E+03  0.35039E+03  0.35788E+02 -0.26622E+01  0.51967E+00  0.51778E+01
  0.13499E+03  0.37840E+03  0.39756E+02 -0.30632E+01  0.53328E+00  0.66967E+01
  0.14482E+03  0.40636E+03  0.44165E+02 -0.34628E+01  0.50778E+00  0.86569E+01
  0.15465E+03  0.43429E+03  0.49049E+02 -0.38609E+01  0.44017E+00  0.11092E+02
  0.16448E+03  0.46216E+03  0.54448E+02 -0.42575E+01  0.32693E+00  0.14043E+02
  0.17431E+03  0.48999E+03  0.60399E+02 -0.46526E+01  0.16487E+00  0.17545E+02
  0.18414E+03  0.51777E+03  0.66835E+02 -0.50462E+01 -0.40147E-01  0.21532E+02
  0.19398E+03  0.54554E+03  0.73533E+02 -0.54391E+01 -0.26830E+00  0.25781E+02
  0.19889E+03  0.55942E+03  0.76911E+02 -0.56355E+01 -0.38501E+00  0.27935E+02
! TAIL COORDINATES & DEFLECTIONS
!   X1           Y1           Z1           DX1           DY1           DZ1
  0.52172E+03  0.13947E+02  0.23139E+03 -0.22407E-02 -0.48059E-01 -0.73073E-01
  0.49440E+03  0.41570E+02  0.21326E+03 -0.22645E-01 -0.41548E+00 -0.62664E+00
  0.46706E+03  0.68878E+02  0.19468E+03 -0.69160E-01 -0.10974E+01 -0.16406E+01
  0.43969E+03  0.96007E+02  0.17585E+03 -0.14674E+00 -0.19583E+01 -0.28915E+01
  0.41228E+03  0.12318E+03  0.15715E+03 -0.25839E+00 -0.27788E+01 -0.40250E+01
  0.38484E+03  0.15057E+03  0.13886E+03 -0.40636E+00 -0.33722E+01 -0.47403E+01
  0.35736E+03  0.17830E+03  0.12117E+03 -0.59225E+00 -0.36308E+01 -0.48636E+01
  0.32984E+03  0.20644E+03  0.10418E+03 -0.81781E+00 -0.34851E+01 -0.42810E+01
  0.30228E+03  0.23502E+03  0.87965E+02 -0.10864E+01 -0.28936E+01 -0.29216E+01
  0.27466E+03  0.26404E+03  0.72520E+02 -0.14043E+01 -0.18686E+01 -0.79512E+00
  0.24699E+03  0.29326E+03  0.57495E+02 -0.17813E+01 -0.63716E+00  0.17521E+01
  0.21926E+03  0.32215E+03  0.42051E+02 -0.22225E+01  0.26801E+00  0.38792E+01
  0.20537E+03  0.33638E+03  0.34001E+02 -0.24625E+01  0.49591E+00  0.46150E+01
```

Fig. 11. Output file DEFLECT.OUT1 for sample case.

TITLE:Beam Skin Thickness and Weight
SUBTITLE:(T and W for 1 side) Tmin = .0400 in.
XLABEL:Distance From Wing Or Tail Root (in.)
YLABEL:Thickness (in.), Weight (lb./in.)
head: Y ~~tweb tcap Wweb Wcap~~ (t_x t_z Dcap1 Dcap2 A_1/A_{tot})
Legend:Wing

0.14950E+02	0.12708E+01	0.14346E+00	0.25679E+01	0.14494E+01
0.44850E+02	0.12371E+01	0.10004E+00	0.24041E+01	0.97207E+00
0.74750E+02	0.11897E+01	0.57942E-01	0.22203E+01	0.54066E+00
0.10465E+03	0.10278E+01	0.40197E-01	0.18387E+01	0.35955E+00
0.13455E+03	0.83495E+00	0.40046E-01	0.14292E+01	0.34274E+00
0.16445E+03	0.72255E+00	0.40000E-01	0.11810E+01	0.32691E+00
0.19435E+03	0.69048E+00	0.40000E-01	0.10753E+01	0.31146E+00
0.22425E+03	0.63884E+00	0.40000E-01	0.94553E+00	0.29602E+00
0.25415E+03	0.74488E+00	0.40227E-01	0.10450E+01	0.28217E+00
0.28405E+03	0.64561E+00	0.10162E+00	0.85585E+00	0.67355E+00
0.31395E+03	0.39435E+00	0.18767E+00	0.49231E+00	0.11714E+01
0.34385E+03	0.40000E-01	0.26367E+00	0.46848E-01	0.15441E+01
0.37375E+03	0.40000E-01	0.22048E+00	0.43759E-01	0.12060E+01
0.40365E+03	0.40000E-01	0.18075E+00	0.40670E-01	0.91887E+00
0.43355E+03	0.40000E-01	0.14167E+00	0.37581E-01	0.66551E+00
0.46345E+03	0.40000E-01	0.10386E+00	0.34492E-01	0.44782E+00
0.49335E+03	0.40000E-01	0.68356E-01	0.31404E-01	0.26833E+00
0.52325E+03	0.40000E-01	0.40000E-01	0.28315E-01	0.14157E+00
0.55315E+03	0.40000E-01	0.40000E-01	0.25226E-01	0.12613E+00
0.58305E+03	0.40000E-01	0.40000E-01	0.22137E-01	0.11068E+00

end
Legend:Tail

0.21431E+02	0.40000E-01	0.28882E+00	0.35301E-01	0.12745E+01
0.64294E+02	0.53791E-01	0.22500E+00	0.45038E-01	0.94195E+00
0.10716E+03	0.20143E+00	0.13369E+00	0.15954E+00	0.52942E+00
0.15002E+03	0.38137E+00	0.54145E-01	0.28479E+00	0.20217E+00
0.19288E+03	0.22806E+00	0.16741E+00	0.15999E+00	0.58720E+00
0.23575E+03	0.18627E+00	0.24114E+00	0.12224E+00	0.79123E+00
0.27861E+03	0.18551E+00	0.29317E+00	0.11334E+00	0.89561E+00
0.32147E+03	0.22409E+00	0.31811E+00	0.12677E+00	0.89982E+00
0.36433E+03	0.30779E+00	0.30306E+00	0.16019E+00	0.78868E+00
0.40720E+03	0.46031E+00	0.21990E+00	0.21874E+00	0.52249E+00
0.45006E+03	0.62138E+00	0.15546E+00	0.26716E+00	0.33421E+00
0.49292E+03	0.57101E+00	0.49803E+00	0.21967E+00	0.95795E+00

end

Fig. 12. Output file THICK.OUT1 for sample case.

$$\mathrm{Dcap1} = \sqrt{A_1}$$
$$\mathrm{Dcap2} = \sqrt{A_2}$$
$$A_{tot} = A_1 + A_2$$

```
!Joined wing geometry
!  Wing span (ft.) =  93.30  :  Tail span (ft.) =  55.98
!  Wing dihedral =  5.000    :  Tail dihedral =  -32.12
!  Wing sweep =  20.00       :  Tail sweep =  -39.55
!  Xtotal =  37.52 : Xwing =  2.521 : Xtail =  35.00
! Coordinates of wing and tail in inches.
!  #      X            Y             Z
!       xWRQC        yWRQC         zWRQC
   1 0.000000E+00 0.000000E+00 0.000000E+00
!       xTTQC        yTTQC         zTTQC
   2 0.207830E+03 0.335880E+03 0.293857E+02
!       xTRQC        yTRQC         zTRQC
   3 0.535370E+03 0.000000E+00 0.240246E+03
!       xWRLE        yWRLE         zWRLE
   4 -.456544E+02 0.000000E+00 0.000000E+00
!       xWTLE        yWTLE         zWTLE
   5 0.193115E+03 0.559800E+03 0.489762E+02
!       xWTTE        yWTTE         zWTTE
   6 0.238770E+03 0.559800E+03 0.489762E+02
!       xWRTE        yWRTE         zWRTE
   7 0.136963E+03 0.000000E+00 0.000000E+00
!       xTRLE        yTRLE         zTRLE
   8 0.510912E+03 0.000000E+00 0.240246E+03
!       xTTLE        yTTLE         zTTLE
   9 0.198047E+03 0.335880E+03 0.293857E+02
!       xTTTE        yTTTE         zTTTE
  10 0.237179E+03 0.335880E+03 0.293857E+02
!       xTRTE        yTRTE         zTRTE
  11 0.608743E+03 0.000000E+00 0.240246E+03
!       xWTQC        yWTQC         zWTQC
  12 0.204529E+03 0.559800E+03 0.489762E+02
end
 ! Connected points
 4    7
 5    6
 4    5
 7    6
 8    11
 9    10
 8    9
 11    10
 1    12
 3    2
```

Fig. 13. Output file XYZ.OUT1 for sample case

```
TITLE:Covergence of Structural Weight
SUBTITLE:Tmin =  4.0000E-02
XLABEL:Iteration Number
YLABEL:Wwing,Wtail,Total Weight (lbs)
! Structural weight, wing, tail, & wing+tail
head Iteration#  Wwing  Wtail  Wwing+Wtail
! ncnvrg BeamWt(i,1) BeamWt(i,2) (BeamWt(i,1)+BeamWt(i,2))
    1      5455.3603   2272.4360   7727.7960
    2      2010.6873    906.3826   2917.0700
    3      1726.7790    871.0422   2597.8212
    4      1701.6142    885.1687   2586.7830
    5      1697.9831    893.7255   2591.7090
    6      1696.6273    897.2806   2593.9080
```

Fig. 14. Ouput file WEIGHT.OUT1 for sample case.

```
TITLE:Drag Coefficient vs. Sweep(1)
XLABEL:Sweep(1)
YLABEL:CD
head: CD              Sweep(1)
  0.12560430E-01  0.20000000E+02
  0.12468990E-01  0.25000000E+02
  0.12376860E-01  0.30000000E+02
  0.12287003E-01  0.35000000E+02
  0.12202534E-01  0.40000000E+02
```

Fig. 15. Output file DvsP.OUT for sample case.

```
TITLE:Weight vs. Sweep(1)
XLABEL:Sweep(1)
YLABEL:Wwing, Wtail, Total Weight (lbs.)
head: Wwing           Wtail           Wwing+Wtail       Sweep(1)
  0.16966273E+04  0.89728060E+03  0.25939080E+04  0.20000000E+02
  0.18562142E+04  0.85244920E+03  0.27086633E+04  0.25000000E+02
  0.20808400E+04  0.80887910E+03  0.28897192E+04  0.30000000E+02
  0.23935600E+04  0.76992710E+03  0.31634870E+04  0.35000000E+02
  0.28310192E+04  0.74234802E+03  0.35733671E+04  0.40000000E+02
```

Fig. 16. Output file WvsP.OUT for sample case.

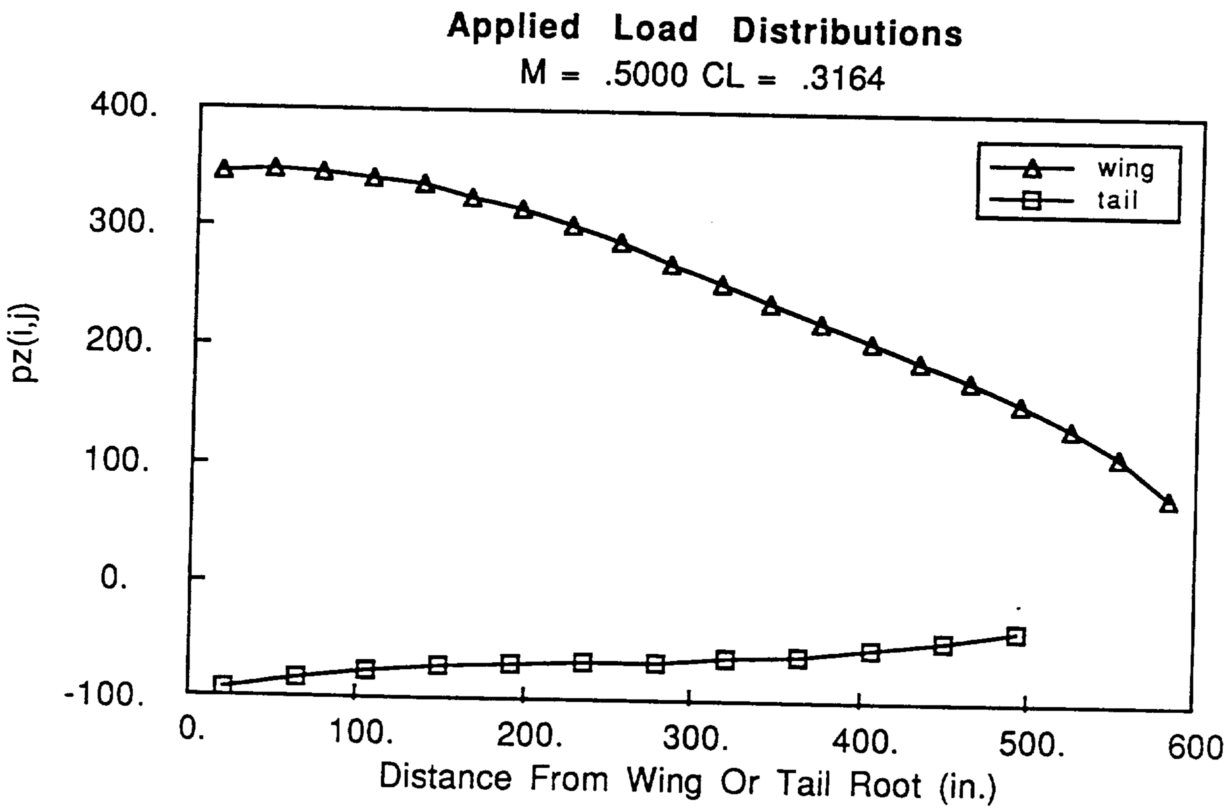

Fig. 17. Applied load distribution in local Z-direction.

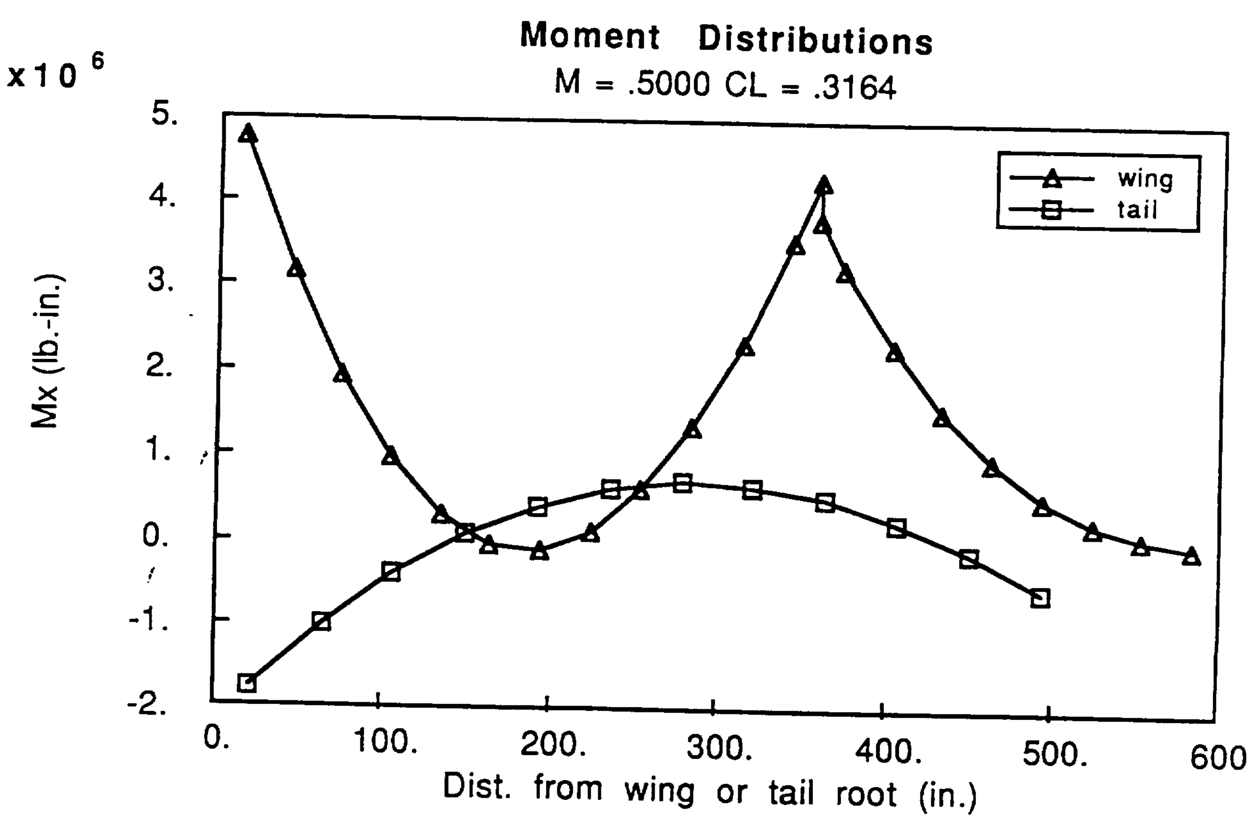

Fig. 18. Moment distribution about the local X-axis.

Appendix Computing Optimal Twist and Loading for Minimum Drag

Assume that the root or tip twist of one surface is denoted ε_i. We note that several quantities depend linearly on ε_i (for small values of ε_i). Denoting aircraft lift coefficient, L, pitching moment coefficient, M, normalwash, at panel k, w_k, and section local lift coefficient, l, we may write:

$$L = \sum_{i=1}^{N} \varepsilon_i L_{,i} \qquad M = \sum_{i=1}^{N} \varepsilon_i M_{,i} \qquad w_k = \sum_{i=1}^{N} \varepsilon_i w_{k,i} \qquad l_k = \sum_{i=1}^{N} \varepsilon_i l_{k,i}$$

where the subscript ',i' denotes the partial derivative with respect to the i^{th} unknown incidence.

The induced drag coefficient is then,

$$D_{ind} = \frac{1}{S_{ref}} \int l(y)\, c \frac{w(y)}{U_\infty} dy \approx \sum_k \frac{c_k \Delta y_k}{S_{ref}} l_k \overline{w}_k$$

and the viscous drag is:

$$D_{visc} = \frac{1}{S_{ref}} \int [C_{d_0} + C_{d_1} l + C_{d_2} l^2]\, c\, dy \approx \sum_k \frac{c_k \Delta y_k}{S_{ref}} [C_{d_0} + C_{d_1} l_k + C_{d_2} l_k^2]$$

Defining the objective function as the total drag plus the lift and moment constraints, with Lagrange multipliers, λ_1 and λ_2:

$$J = D_{ind} + D_{visc} + \lambda_1 (L - L_{ref}) + \lambda_2 (M - M_{ref})$$

At the optimum:

$$\frac{\partial J}{\partial \varepsilon_i} = \frac{\partial J}{\partial \lambda_i} = 0.$$

This represents N+2 equations as follows:

$$J_{,i} = \sum_k \frac{c_k \Delta y_k}{S_{ref}} (l_{k,i} \overline{w}_k + l_k \overline{w}_{k,i}) + \frac{c_k \Delta y_k}{S_{ref}} [C_{d_1} l_{k,i} + 2 C_{d_2} l_k l_{k,i}] + \lambda_1 L_{,i} + \lambda_2 M_{,i} = 0$$

or:

$$J_{,i} = \sum_j \sum_k \frac{c_k \Delta y_k}{S_{ref}} [l_{k,i} \overline{w}_{k,j} + l_{k,j} \overline{w}_{k,i} + 2 C_{d_2} l_{k,j} l_{k,i}] \varepsilon_j + \lambda_1 L_{,i} + \lambda_2 M_{,i} + \sum_k \frac{c_k \Delta y_k}{S_{ref}} C_{d_2} l_{k,i}$$

$$= \sum_j D_{ij} \varepsilon_j + D_i' + \lambda_1 L_{,i} + \lambda_2 M_{,i} = 0$$

And the constraints:

$$J_{,\lambda_1} = L - L_{ref} = \sum_j L_{,j} \varepsilon_j - L_{ref} = 0 \qquad J_{,\lambda_2} = M - M_{ref} = \sum_j M_{,j} \varepsilon_{,j} - M_{ref} = 0$$

This may be written as a linear system of equations for ε_i and the λ's:

$$
\left[\begin{array}{ccc}
 & L_{,i} & M_{,i} \\
D_{ij} & \cdot & \cdot \\
 & \cdot & \cdot \\
\hline
L_{,j} \; \cdot \; \cdot \; \cdot & 0 & 0 \\
M_{,j} \; \cdot \; \cdot \; \cdot & 0 & 0
\end{array}\right]
\left[\begin{array}{c}
\varepsilon_j \\
\cdot \\
\cdot \\
\hline
\lambda_1 \\
\lambda_2
\end{array}\right]
=
\left[\begin{array}{c}
-D_i' \\
\cdot \\
\cdot \\
\hline
L_{ref} \\
M_{ref}
\end{array}\right]
$$

Note that to constrain one of the incidence angles we may add a third Lagrange multiplier, the desired value to the right hand side, a 0 0 1 0 0 ... as the last row of the matrix, and a new column with 0 0 0 0 0 1 in the appropriate row. If the matrix is singular because of too many or two few degrees of freedom, the user must change the inputs.